CANNING COLLEGE

Year 12 Mathematics
MAS 3CD
SEMESTER 2 EXAMINATION 2010

Section One (Calculator Free)

SOLUTIONS

Name: ______________________ **Your Score:** ______________

Lecturer: ______________________

Class Code: ______________________

Question/Answer Booklet

Time allowed for this section

Reading time before commencing work: five minutes

Working time for paper: fifty minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula sheet.

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters.

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

Structure of this examination

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator Free	**8**	**8**	**50**	**40**	
Section Two Calculator Assumed	**13**	**13**	**100**	**80**	
			Total marks	**120**	

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. . Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions in the spaces provided. Answer **all** questions.
   It is recommended you **do not use pencil**, except in diagrams

3. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

4. **Show all working clearly**. Any question, or part question, worth more than 2 marks requires valid working or justification to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

1. [4 marks]

The complex number $z = x + yi$.

Given $\text{Re}\left[\frac{z}{z-2}\right] = 0$ show that $x^2 + y^2 - 2x = 0$, $x \neq 2$

$$\text{Re}\left(\frac{x+yi}{(x+yi)-2}\right) = 0$$

$$\text{Re}\left(\frac{x+yi}{(x-2)+yi} \times \frac{(x-2)-yi}{(x-2)-yi}\right) = 0 \quad ✓✓$$

$$\text{Re}\left(\frac{x(x-2) - xyi + (x-2)yi + y^2}{(x-2)^2 + y^2}\right) = 0 \quad ✓$$

$$\frac{x(x-2) + y^2}{(x-2)^2 + y^2} = 0$$

$$\therefore x^2 + y^2 - 2x = 0 \quad ✓$$

2. [3 marks]

(a) Under what circumstances will $(A+B)(A-B) = A^2 - B^2$ where A and B are 2×2 matrices? [1]

$AB = BA$ ✓

(i.e. A and B are commutative under multiplication)

(b) Write down 2 matrices (A and B) which will make the above equation true if neither matrix is the zero matrix, the identity matrix nor a multiple of the identity matrix. [2]

many possible examples

Eg $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ ✓✓

3. [2 marks]

Determine the following limit:

$\lim_{x \to 0} \quad \frac{k \tan 3x}{x}$ [2]

$$\lim_{x \to 0} \quad \frac{k}{x} \frac{\sin 3x}{\cos 3x}$$

$$= \lim \quad 3k \frac{\sin 3x}{3x \cos 3x}$$

$$= \quad 3k \quad \checkmark\checkmark$$

4. [8 marks]

Find $\frac{dy}{dx}$ for each of the following:

(a) $y = (e^{2x} - 4)^2$ [2]

$$y' = 2(e^{2x} - 4) \cdot (e^{2x} \cdot 2) \quad \checkmark$$

$$= 4e^{2x}(e^{2x} - 4) \quad \checkmark$$

$$= 4e^{4x} - 16e^{2x}$$

(b) $y = \int_5^{3x} 7sin2t \, dt$ [2]

$$7 \sin 6x \quad \times \quad 3 \quad \checkmark$$

$$= 21 \sin 6x \quad \checkmark$$

(c) $6x^2 - 4x \sin y + \cos 4y = 8$ [4]

diff wrt x

$$12x - [4x \cdot \cos y \cdot y' + 4\sin y] - 4\sin y \, y' = 0 \quad \checkmark\checkmark$$

$$y'[-4x \cos y - 4\sin y] = 4\sin y - 12x \quad \checkmark$$

$$y' = \frac{12x - 4\sin y}{4x \cos y + 4\sin y} \quad \checkmark$$

or $y' = \dfrac{3x - \sin y}{x \cos y + \sin y}$

5. [3 marks]

Ben is training for a fun run. If he trains today then the probability that he will train tomorrow is 0.4. If he does not train today, then the probability that he will train tomorrow is 0.75.

(a) Enter the information in the following transition matrix, T

$$\begin{array}{rc} & \begin{array}{cc}\text{Train today} & \text{Don't train today}\end{array} \\ \begin{array}{r}\text{Train tomorrow} \\ \text{Don't train tomorrow}\end{array} & \begin{pmatrix} 0.4 & 0.75 \\ 0.6 & 0.25 \end{pmatrix} = T \end{array}$$

✓✓ −1/error [2]

(b) The probability matrix T^3 is shown below. Describe the meaning of the value which is circled in the matrix.

$$T^3 = \begin{pmatrix} 0.54 & 0.58 \\ 0.46 & 0.42 \end{pmatrix}$$

If Ben trains today, then the probability he will not train in 3 days time is 0.46 ✓ [1]

6. [5 marks]

Use mathematical induction to prove the following conjecture:

$$1 + (1+x) + (1+x)^2 + ... + (1+x)^{n-1} = \frac{(1+x)^n - 1}{x} \quad , \quad n \geq 1$$

If $n = 1$ LHS $= 1$ RHS $= 1$

∴ statement true for $n=1$

Assume statement true for $n = k$

ie. $1 + (1+x) + (1+x)^2 + ... (1+x)^{k-1} = \frac{(1+x)^k - 1}{x}$

now consider $n = k+1$

$1 + (1+x) + (1+x)^2 + ... + (1+x)^{k-1} + (1+x)^k$

$= \frac{(1+x)^k - 1}{x} + (1+x)^k$

$= \frac{(1+x)^k - 1}{x} + \frac{(1+x)^k x}{x}$

$= \frac{(1+x)^k [(1+x)] - 1}{x}$

$= \frac{(1+x)^{k+1} - 1}{x}$

Hence, by induction, the conjecture is true.

7. [7 marks]

If $a = 2e^{\frac{-\pi i}{4}}$ and $b = e^{\frac{\pi i}{3}}$ find

(a) $\frac{a}{b}$ in polar form [3]

$$\frac{a}{b} = \frac{2e^{-\pi i/4}}{e^{\pi i/3}}$$

$$= 2e^{-\frac{7\pi}{12}i} \quad \checkmark\checkmark$$

$$= 2\,\text{cis}\left(-\frac{7\pi}{12}\right) \quad \checkmark$$

(b) a^3 in Cartesian form [3]

$$a^3 = \left(2e^{-\frac{\pi i}{4}}\right)^3$$

$$= 8\,\text{cis}\left(-\frac{3\pi}{4}\right) \quad \checkmark$$

$$= 8\cos\left(-\frac{3\pi}{4}\right) + 8\sin\left(-\frac{3\pi}{4}\right)$$

$$= -\frac{8}{\sqrt{2}} - \frac{8}{\sqrt{2}}i \quad \checkmark\checkmark$$

(c) $|b|$ [1]

$$|b| = 1 \quad \checkmark$$

8. [8 marks]

Determine the following integrals, writing your answers in simplified form.

(a) $\int \cos 2t \sin^5 2t \quad dt$ [2]

$= \frac{1}{12} \sin^6 2t + c$ ✓✓

(b) $\int \frac{4+4cosx}{x+sinx}\, dx$ [2]

$4 \ln |x + \sin x| + c$ ✓✓

(b) $\int_0^4 \frac{x}{\sqrt{25-x^2}}\, dx$ let $u = 25 - x^2$ [4]

$u = 25 - x^2$

$\frac{du}{dx} = -2x$ ✓

$x = 0 \quad u = 25$

$x = 4 \quad u = 9$

$= \int_{25}^{9} \frac{x}{\sqrt{25-x^2}} \cdot \frac{1}{-2x}\, du$

$= -\frac{1}{2}\int_{25}^{9} \frac{1}{\sqrt{u}}\, du$ ✓

$= -\frac{1}{2}\left[2\sqrt{u}\right]_{25}^{9}$ ✓

$= -\frac{1}{2}\left[6 - 10\right]$

$= 2$ ✓

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Year 12 Mathematics
MAS3CD
SEMESTER 2 EXAMINATION 2010

Section Two (Calculator Assumed)

Name: ______________________ **Your Score:** Solutions

Lecturer: ______________________

Class Code: ______________________

Question/Answer Booklet

Time allowed for this section

Reading time before commencing work:	10 minutes
Working time for paper:	100 minutes
Total Marks for this section:	**80**

Material required/recommended for this section

To be provided by the supervisor

Question/answer booklet for Section Two.

Formula sheet.

To be provided by the candidate

Drawing instruments, templates, notes on two unfolded sheets of A4 paper and calculators satisfying the conditions set by the Curriculum Council for this subject

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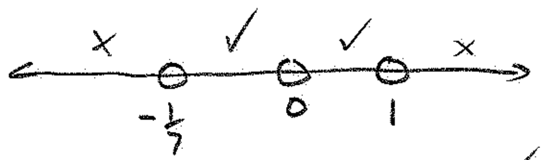
1. [4 marks]

Solve algebraically $\left|\frac{3x+1}{x}\right| \geq 4$ $\qquad x \neq 0$

Let $\frac{3x+1}{x} = 4$ or $\frac{3x+1}{x} = -4$

$3x+1 = 4x$ or $3x+1 = -4x$

$x = 1$ $\qquad x = -\frac{1}{7}$ ✓✓



✓ ✓

$\therefore -\frac{1}{7} \leq x < 0, \quad 0 < x \leq 1$

2. [3 marks]

(a) Find the equation of the plane passing through the points A(1,2,3) and B(3,-2,1) and perpendicular to the plane 3x – 2y + 4z = 5
Give you answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. [3]

Let $P = 3x - 2y + 4z = 5$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} = 5$

$\therefore \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ is normal to plane. P ✓

$\therefore \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ is parallel to required plane

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ ✓

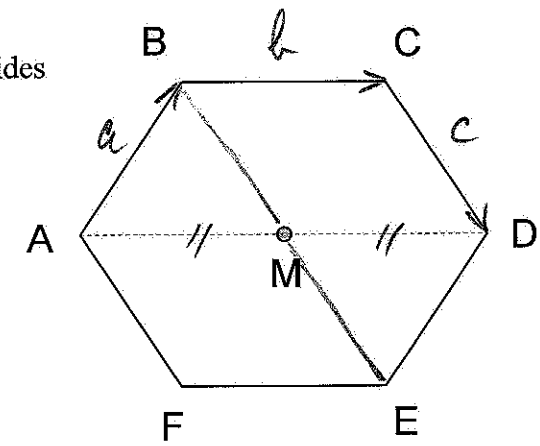
$\therefore \underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$

or $r = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ ✓

3. [5 marks]

Let ABCDEF be a regular hexagon
(i.e. all sides are of equal length, opposite sides parallel and all angles are of equal size).
M is the midpoint of AD.

Let $\overrightarrow{AB} = \boldsymbol{a}$, $\overrightarrow{BC} = \boldsymbol{b}$, $\overrightarrow{CD} = \boldsymbol{c}$



Use vector methods to prove that $\overline{BE}$ passes through point M where M is the midpoint of $\overline{BE}$.

$$\overrightarrow{AD} = \underset{\sim}{a} + \underset{\sim}{b} + c$$

$$\overrightarrow{AM} = \tfrac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c})$$

$$\overrightarrow{BM} = -\underset{\sim}{a} + \tfrac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c})$$

$$= -\tfrac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c})$$

✓✓✓✓

$$\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$$

$$= \underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{a}$$

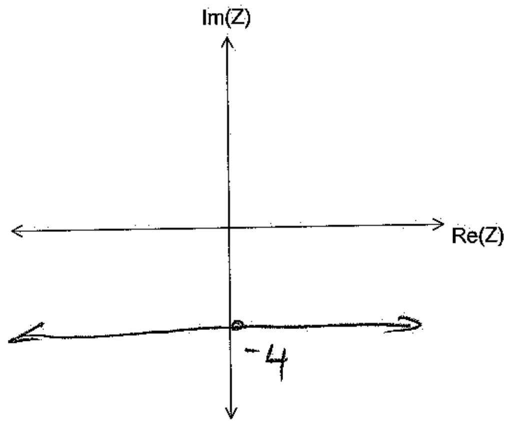
$$= 2\overrightarrow{BM}$$

Hence M is mid-point of $\overrightarrow{BE}$

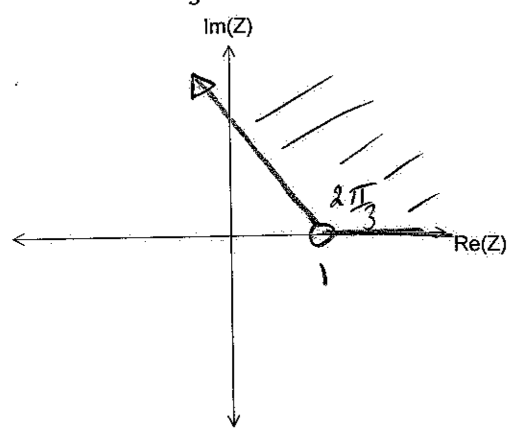
4. [6 marks]

Show each of the following sets of points on the Argand Plane.

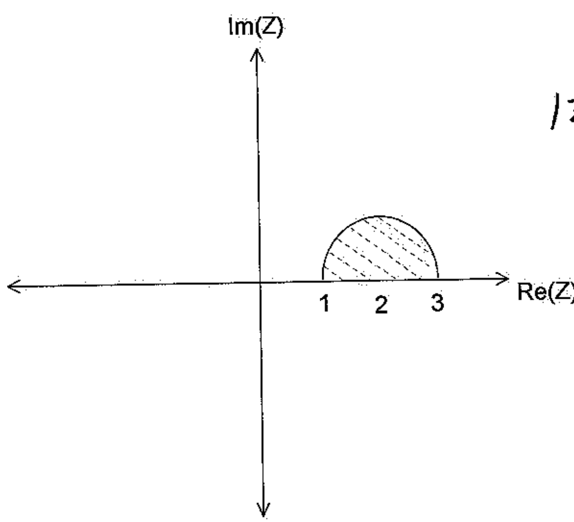
(a) (i) $Im(\bar{Z}) = 4$ [2]



(ii) $0 \leq \text{Arg}(Z - 1) \leq \frac{2\pi}{3}$ [2]



(b) Write down two complex inequalities which when graphed will result in the region shown in the following diagram. [2]



$|z-2| \leq 1$ and

$Im(z) \geq 0$

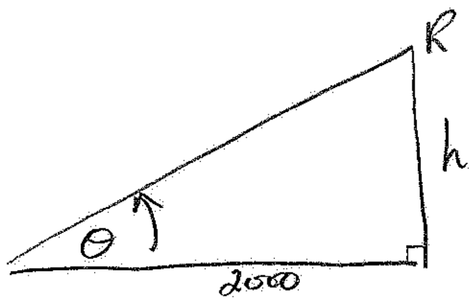
(OR $Arg(z) \geq 0$)

5. [5 marks]

A television camera is positioned 2000 metres from a rocket which is about to be launched.
The rocket rises vertically and the height, h, reached during the first 10 seconds of its flight is given by

$$h = 2t^3 + 40t \quad \text{metres}$$

If the camera remains fixed on the base of the rocket, how fast is the camera's angle of elevation changing when the rocket is 1 kilometre high?



$$h = 2t^3 + 40t$$

$$\frac{dh}{dt} = 6t + 40$$

$$\tan\theta = \frac{h}{2000}$$

diff wrt t

$$\frac{1}{\cos^2\theta} \times \frac{d\theta}{dt} = \frac{1}{2000} \times \frac{dh}{dt}$$

$$\frac{1}{\cos^2\theta} \times \frac{d\theta}{dt} = \frac{1}{2000} \times (6t + 40)$$

when $h = 1000$ $\quad \cos\theta = \frac{2000}{\sqrt{5\,000\,000}}$ and $t = 7.1005$

$$\frac{d\theta}{dt} = \frac{4\,000\,000}{5\,000\,000} \times \frac{1}{2000} \times (6(7.1005) + 40)$$

$$= 0.033 \text{ radians/sec}$$

6. [9 marks]

A hot air balloon takes off 20 km north of a country town called York.
The velocity of the balloon is given by the vector $5\boldsymbol{i} - 16\boldsymbol{j} + 0.4\boldsymbol{k}$ km/hr, where the vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors in the direction east, north and vertically upward respectively.

(a) Write down the initial position vector of the balloon. [1]

$\langle 0, 20, 0 \rangle$ ✓

(b) Write down the position vector of the balloon after 30 minutes. [2]

$\langle 0, 20, 0 \rangle + 0.5\langle 5, -16, 0.4 \rangle$ ✓

$= \langle 2.5, 12, 0.2 \rangle$ ✓

(c) Determine the speed the balloon is moving. [2]

Speed $= \sqrt{5^2 + (-16)^2 + (0.4)^2}$ ✓

$= 16.77$ km/hr. ✓

(d) (i) If the balloon leaves the ground at 6 am, at what time will the balloon be closest to York?

position of balloon $= \langle 5t, 20-16t, 0.4t \rangle$

" " York $= \langle 0, 0, 0 \rangle$

$\therefore$ Distance to York $= \sqrt{(5t)^2 + (20-16t)^2 + (0.4t)^2}$ ✓

From a calculator ✓

distance a min at $t = 1.138$ hrs

i.e. 7:08 ✓

(ii) What is the closest distance the balloon comes to York? [4]

5.98 km ✓

7. [6 marks]

(a) If $A = \begin{bmatrix} 0 & 4 & 2 \\ 2 & 2 & 2 \\ -2 & -4 & -4 \end{bmatrix}$ find A^2 [1]

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad \text{or} \quad 4I$$

(b) Use the result from part (a) to solve the following simultaneous equations.

$$\begin{aligned} 4y + 2z &= -2 \\ 2x + 2y + 2z &= 0 \\ 2x + 4y + 4z &= -6 \end{aligned}$$

[3]

$$\begin{pmatrix} 0 & 4 & 2 \\ 2 & 2 & 2 \\ -2 & -4 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 6 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} 0 & 4 & 2 \\ 2 & 2 & 2 \\ -2 & -4 & -4 \end{pmatrix}^2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 4 & 2 \\ 2 & 2 & 2 \\ -2 & -4 & -4 \end{pmatrix} \begin{pmatrix} -2 \\ 0 \\ 6 \end{pmatrix}$$

$$4I \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \\ -20 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix} \checkmark \quad \therefore \quad x = 3, \; y = 2, \; z = -5$$

(b) Under what conditions is the matrix $\begin{bmatrix} a & -1 \\ -2 & a-1 \end{bmatrix}$ singular? [2]

$$a(a-1) - (-2)(-1) = 0$$

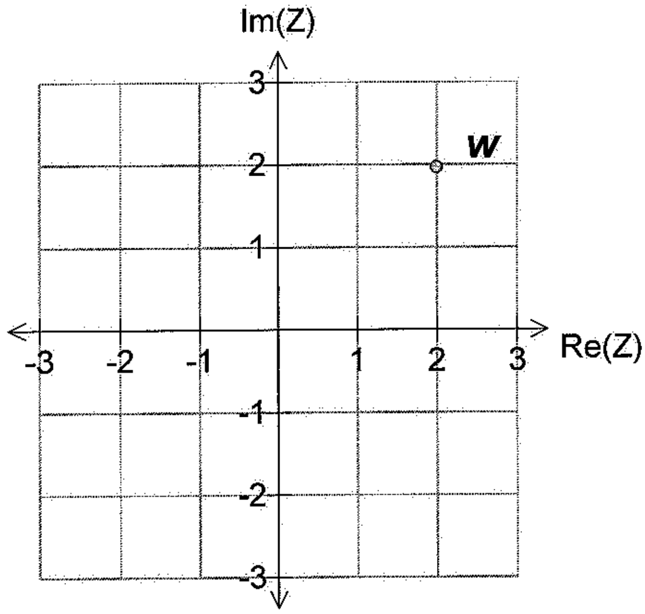
$$a^2 - a - 2 = 0$$

$$(a+1)(a-2) = 0$$

$$\therefore \; a = 2 \text{ or } a = -1$$

8. [5 marks]

The Argand plane below shows a complex number W which is **one** of the **three** roots of Z.



(a) Write down the 3 roots of Z in polar form giving the arguments in degrees. [2]

$$\left.\begin{aligned}\sqrt[3]{Z} &= \sqrt{8}\text{ cis } 45^\circ\\ &\quad\sqrt{8}\text{ cis } 165^\circ\\ &\quad\sqrt{8}\text{ cis }(-75^\circ)\end{aligned}\right\} -1\text{ /error.}$$

(b) Find Z in Cartesian form. [2]

$$Z = \left[\sqrt{8}\text{ cis } 45^\circ\right]^3$$
$$= 16\sqrt{2}\text{ cis } 135^\circ$$
$$= -16 + 16i$$

(c) Determine the sum of the 3 roots. [1]

0

9. [8 marks]

A large chicken is taken from the refrigerator. The temperature of the chicken is 3°C when it is placed into an oven which has been preheated to a temperature of 180°C. The temperature, T°C , of the chicken after t minutes in the oven is given by the equation:

$$\frac{dT}{dt} = -k(T - 180).$$

(a) Use a calculus method to show that $T = 180 - 177e^{-kt}$ satisfies the conditions given and hence determine the value of k. [4]

$$\frac{1}{T-180} \frac{dT}{dt} = -k$$

$$\frac{1}{180-T} \frac{dT}{dt} = k \quad \checkmark$$

$$\int \frac{1}{180-T} \frac{dT}{dt}\, dt = \int k\, dt$$

$$-\ln(180-T) = kt + c \quad \checkmark$$

$$180 - T = e^{-kt} \cdot e^{c} \quad \checkmark$$

$$T = 180 - e^{-kt} \cdot e^{c}$$

$$\text{at } t = 0 \quad T = 3 \quad \therefore e^{c} = 177 \quad \checkmark$$

$$\therefore T = 180 - 177e^{-kt}$$

(b) The chicken is placed in the oven at 6pm and by 7pm has reached a temperature of 70°C. If the chicken is cooked once its temperature reaches 85°C , at what time will the chicken be cooked? Give your answer to the nearest minute. [4]

$$\text{If } t = 1 \quad T = 70$$

$$\therefore 70 = 180 - 177e^{-k(1)} \quad \checkmark$$

$$\therefore k \approx 0.4757 \quad \checkmark$$

$$85 = 180 - 177e^{-0.4757(t)}$$

$$t = 1.308 \quad \checkmark$$

Chicken cooked at ≈ 7:18 ✓

10. [8 marks]

The matrix $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ maps the point A (5, 2) onto the point A′ (3, 1) and the point B (2, 1) on to the point B′ (2, -2).

(a) Determine matrix M. Show your method. [2]

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}^{-1} \checkmark$$

$$= \begin{pmatrix} -1 & 4 \\ 5 & -12 \end{pmatrix} \checkmark$$

(b) The area of triangle ABC is 10 square units. Determine the area of triangle A′B′C′ under the transformation in part (a). [2]

$$\text{Det} \begin{pmatrix} -1 & 4 \\ 5 & -12 \end{pmatrix} = -8 . \checkmark$$

$\therefore$ Area $\triangle A'B'C' = |-8| \times 10 = 80$ sq units. ✓

The points A′ and B′ are then transformed to A″ and B″ using a rotation of 90° clockwise about the origin.

(c) Determine the coordinates of A″ and B″. [2]

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -3 & -2 \end{pmatrix} \checkmark$$

$\therefore$ $A''(1, -3)$ $B''(-2, -2)$ ✓

(d) Write down a single matrix which will map the points A and B onto A″ and B″. [2]

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ 5 & -12 \end{pmatrix} = \begin{pmatrix} 5 & -12 \\ 1 & -4 \end{pmatrix}$$

✓ ✓

11. [9 marks]
A farmer is breeding freshwater crayfish (called yabbies) in one of his dams. He has collected the following data on their breeding and survival rates.

Age (years)	1	2	3	4
Population	750	1200	900	600
Birth Rate	0	0.7	1.4	0.5
Survival Rate	0.7	0.6	0.5	0

(a) Construct a Leslie matrix, L, to represent the information given in the table. [2]

$$L = \begin{pmatrix} 0 & 0.7 & 1.4 & 0.5 \\ 0.7 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{pmatrix} \quad ✓✓$$

(b) Use the current population figures to predict the population in each age group in four years time. [2]

$$L^4 \begin{pmatrix} 750 \\ 1200 \\ 900 \\ 600 \end{pmatrix} = \begin{pmatrix} 2274 \\ 1257 \\ 672 \\ 504 \end{pmatrix} \quad ✓✓$$

mat B

(c) Determine the percentage growth rate in the population between the 10th year and the 11th year into the future. [2]

$$(1\ 1\ 1\ 1)\ L^{10} \times B = 6631$$
$$(1\ 1\ 1\ 1)\ L^{11} \times B = 7040 \quad ✓$$

$\therefore$ % growth $\approx 6.2\%$ ✓

(d) The farmer wishes to harvest 40% of the two year old crayfish each year. Will the farmer be able to maintain this harvesting level in the long term. Justify your answer with appropriate matrix calculations. [3]

new Leslie matrix $\begin{pmatrix} 0 & 0.42 & 1.4 & 0.5 \\ 0.7 & 0 & 0 & 0 \\ 0 & 0.36 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{pmatrix} = M$

$$(1\ 1\ 1\ 1)\ M^{20} \times B = 81$$
$$(1\ 1\ 1\ 1)\ M^{40} \times B = 22$$

Harvest rate cannot continue.

12. [4 marks]

Consider the function $P = 2\pi\sqrt{\frac{t}{5}}$

Use a calculus method to determine the error in calculating P if t is measured to be 3 ± 0.1.

$$\frac{\delta P}{\delta t} \approx \frac{dP}{dt}$$

$$\delta P \approx (0.1)\ 2\pi \frac{1}{2\sqrt{5t}}$$

✓ ✓

If $t = 3$

$$\delta P \approx 0.1 \frac{\pi}{\sqrt{15}}$$

$$\approx 0.08$$

ie. error in calculating P is $\underline{0.08}$

(or ± 0.08)

13. [8 marks]

An object is moving along the x axis such that its velocity after t seconds is given by

$$v = 2\pi \cos 4\pi t + 4\pi \cos 2\pi t$$

Given the object is initially at $x = 4$, determine

(a) The maximum velocity of the object. [1]

$t = 0 \quad v = 2\pi + 4\pi$

$= 6\pi$ ✓

(b) the time taken for the object to return to its starting position for the first time. [3]

$x = \frac{1}{2}\sin 4\pi t + 2\sin 2\pi t + c$ ✓

at $t = 0$ $x = 4$ $\therefore c = 4$ ✓

$x = \frac{1}{2}\sin 4\pi t + 2\sin 2\pi t + 4$

$\therefore$ Returns to $x = 4$ when $t = \frac{1}{2}$ sec ✓

(c) the distance the object travels in the first 0.1 second.
(Use your calculator but indicate the method used) [2]

distance in 8 sec $= \int_0^{0.1} |v|\, dt$

$= 1.65$ m ✓✓

(d) the acceleration of the object at $t = 2$ seconds. [2]

$a = -8\pi^2 \sin 4\pi t - 8\pi^2 \sin 2\pi t$ ✓

$a(2) = 0$ ✓

END OF PAPER